On the Mobile Wireless Access via MIMO Relays

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***Abstract*—While a network with stationary nodes can provide large capacity in many current wireless systems, a network with mobile users suffers from severe performance degradation. This happens due to quick channel variation and its resulting protocol overheads that are especially large for multiuser multiple-input multiple-output (MIMO) systems. In this paper, we propose a relay-assisted multiuser MIMO downlink system to support highly mobile wireless access. Through a theoretical throughput analysis that explicitly considers the protocol overheads, we show that the MIMO relay significantly enhances the sum throughput of the system.**

I. Introduction

It is believed that next generation wireless networks will adopt multiple-input multiple-output (MIMO) technology to have higher speed and larger capacity than its predecessors. Through signal processing over multiple radio chains, MIMO opens a new dimension–*spatial domain*–in addition to the traditional temporal, spectral resources, greatly enhancing the system capacity. As a result, data delivery such as high definition video streaming over wireless is being discussed even for a cellular like highly loaded system.

Considering different hardware complexities of user equipment and a base station (BS), multiuser MIMO[1] is regarded as one of the core technologies for downlink (a BS to user link) in most wireless standardization activities [1], [2]. Multiuser MIMO is especially helpful to improve the sum throughput of the system when there exists asymmetry between the numbers of transmit and receive antennas [3].

Despite its potential, there is a critical drawback that multiuser MIMO downlink requires channel state information (CSI) with high accuracy. This leads physical and medium access control (MAC) layer protocols to consume a part of given resources as *protocol overheads*. The minimum protocol overheads are a pilot signal from the transmitter for each transmit antenna and optional CSI feedback from the receivers, the amount of which scales with the number of antennas and users. Consequently, it is widely accepted that the system performance enhanced by multiuser MIMO would be limited only to a network with stationary users and having more antennas at user equipment is believed to be essential to support high mobility.

To support mobile wireless access even with a small number of user antennas, we propose a relay-assisted multiuser MIMO

[1]Multiuser MIMO refers only to the MIMO broadcast channel in this paper.

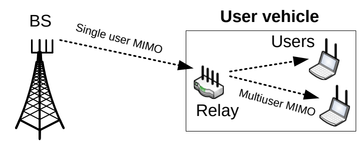

Fig. 1. The proposed relay-assisted MIMO network.

downlink illustrated in Fig. 1. In our system, a BS has multiple antennas, and users have fewer antennas than BS's. A MIMO relay, which has multiple antennas, is installed in a user vehicle to compensate the performance loss from the high mobility. Consequently, while stationary users are directly served by the BS without any relay, the mobile users take advantage of the relay in the vehicle. The downlink for the mobile users is thus composed of two hops: BS-relay and relay-user links. The relay decodes the signal from the BS, encodes it again and forwards to the users. The physical volume of the vehicle provides large space for mounting many antennas at a relay.

Placement of a MIMO relay at a user vehicle creates two asymmetric channels which can be exploited to substantially enhance the sum throughput of the system; (i) the BS-relay link has short coherence time, which is not appropriate to use multiuser MIMO downlink transmission due to feedback protocol overheads, but more suitable to do single user MIMO. (ii) The relay-user link has high channel gain and is stationary, being proper to use multiuser MIMO. Moreover, the relay-user link is barely interfered by the transmission from the BS, which achieves spatial reuse between the BS-relay and relay-user links. Overall, the proposed system is shown to outperform multiuser MIMO downlink system without a relay and achieves high throughput even with high mobility.

Previous Work: The physical layer techniques for two hop MIMO relaying are discussed in [4], [5], focusing more on throughput performance. The authors [4] show that decoding-and-forwarding relay outperforms amplifying-and-forwarding and hybrid relays when MIMO relays employ one of single user MIMO schemes with CSI at the transmitter for free. Amplifying-and-forwarding MIMO relays are optimized via geometric programming in [5], utilizing multi-antenna capability of the relay to concurrently serve users by multiuser MIMO, which closely resembles our architecture.

In contrast, we assume a decoding-and-forwarding MIMO relay that is mobile with the users. This architecture is mentioned in IEEE 802.16j [6]. However, our work distinguishes itself from the previous work in two viewpoints. First, the performance of the MIMO relay is analyzed with an explicit protocol overhead model for single and multiuser MIMO. Second, through the analysis, we have found that the time-sharing for two hop relaying can be minimized by scheduling the relays for spatial reuse.

Organization: In Section II, the proposed system and analytic model are explained. Section III provides the theoretic throughput of the multiuser MIMO downlink and proposed relay-assisted system. Numerical results are analyzed in Section IV. Section V summarizes the open challenges to achieve theoretic gains in practice. This paper finally concludes in Section VI.

II. Proposed System and Analytic Model

We propose a relay-assisted multiuser MIMO downlink for mobile users as shown in Fig. 1. A BS has many antennas and a relay has multiple antennas so that single user MIMO can be used for the BS-relay downlink. In addition, users have a single antenna. The only requirement for the proposed system is that users have less antennas than BS where multiuser MIMO downlink might have been adopted. A relay is installed at a user vehicle, thus having high mobility as users do. We assume that the wireless communication in this system is scheduled and coordinated in time and spatial domains within a narrow bandwidth.

A. Channel Characteristics

Interestingly, the BS-relay and relay-user links have radically different characteristics. The channel response of the BS-relay link significantly varies over time while that of the relay-user link is stationary. In addition, because of their proximity, the channel gain of the relay-user link is very high, compared to the BS-relay link.

For the purpose of throughput analysis, it is assumed that the signal attenuates in distance by free space path-loss model with exponent 3.[2] For a given received SNR from the BS to users, the distance between the BS and the users is computed by the free space model. The relay is placed 1 meter apart from the users on the line between the BS and users. For fast fading, Rayleigh fading model is used with block fading law. The channel varies due to fast fading independently, discretely at the beginning of each mini-slot (will be defined shortly).

B. MIMO Schemes

We adopt zero-forcing receiver and zero-forcing beamforming [7] for the BS-relay link and relay-user links, respectively. In the standards activities [1], [2], these two are being considered due to their simplicity in implementation. Note that unlike the relay-user link, we do not apply any preprocessing on the BS-relay link in order to avoid the need for the CSI feedback overheads. For comparison, we consider the base-line system where the BS directly serves the users via multiuser MIMO by zero-forcing beamforming.

C. Frame Format

It is assumed that the transmission from the BS to the users through the relay follows a frame format illustrated in Fig. 2. According to individual mobility, each set of user and relay has a different time duration of which a channel response lasts flat, namely coherence time. This time duration is termed a mini-slot. In general, the duration for a BS-relay mini-slot is far shorter than that of relay-user's since the relay and users move together.

Each mini-slot is used in part to exchange necessary information, which corresponds to pilots and (channel vector) feedback, and the rest of it is used for actual data transmission. For a BS-relay mini-slot, there are only pilots without feedback, thus performing open-loop transmission.[3] For a mini-slot of the relay-user link, however, both pilots and feedback exist since the zero-forcing beamforming relies on the CSI at the transmitter.

Data is forwarded through the BS-relay link using a BS-relay super-slot, which is composed of multiple mini-slots. The forwarded data is delivered to users by using a relay-user super-slot, which is a multiple of a relay-user mini-slot. A relay-user super-slot immediately follows BS-relay's, forming an entire single frame. Note that the transmission efficiency is maximized when the throughput during a BS-relay super-slot is the same as throughput by the relay-user.

The duration of a time slot and frame is measured by a symbol unit, and it is fixed throughout this paper. As a result, throughput is also measured by bits per symbol, which is given by Shannon's equation: $\log(1+\mathsf{SNR})$ where SNR is signal-to-noise-ratio. A symbol thus represents practical limitations on time sampling rate and frequency band allocation of a system. Most of all, the overheads are modeled in a concise way when the symbol unit is employed.

D. Overhead Model

The authors in [8] showed that the training overheads (pilots) can be minimally reduced down to one symbol per transmit antenna. For feedback, our system adopts the random vector quantization to compress channel direction information (CDI), which is shown to perform close to the optimal codebook [9]. Though channel magnitude should be fed back to complement the CSI at the transmitter, only CDI feedback is taken into account for our overhead model due to the insignificant of the amount of feedback for the magnitude information.

Denote the number of antennas at the BS by N_{bs} at the relay by N_{rn} and at the user k by N_k, which is assumed N_k for all k. There are always N_{bs} users associated with the relay. Also denote the protocol overheads of single user MIMO with zero-forcing receiver by V_{su} and the overheads of the multiuser

[2]A path-loss exponent does not affect the tendency of the curves in the figure in Section III. In fact, the gain goes higher when the exponent gets larger.

[3]Even in this case, the rate information should be sent back to the BS. However, this part is insignificant compared to pilot and channel vector feedback, thus being ignored.

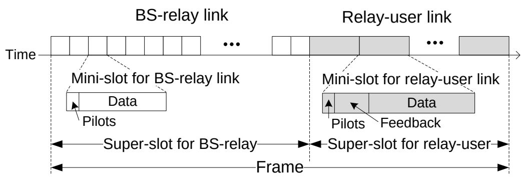

Fig. 2. Mini-slots, super-slots and frame for transmission. Due to vehicle's mobility, mini-slot for the BS-relay link is shorter than relay-user's.

MIMO downlink with zero-forcing beamforming by V_{mu}. Two quantities per mini-slot are then easily computed by

$$V_{su} = N_{bs}, \tag{1}$$

where there is pilot overhead only, and

$$V_{mu}^{(B)} = N_{tx} + K \cdot \lceil B\{\log(1+N_{tx}\mathsf{SNR})\}^{-1}\rceil, \tag{2}$$

where B is the number of bits for CDI feedback per user and the feedback rate per symbol, $\log(1+N_{tx}\mathsf{SNR})$, is the throughput achievable by maximum ratio combining in single-input multiple-output transmission. N_{tx} is the number of transmit antennas, which is N_{rn} for the relay-user link in the proposed system and is N_{bs} for the baseline system. K is the number of users associated with either the relay in ours or the BS in the baseline, which is N_{bs} for both cases.

III. Throughput Analysis

To see the theoretical gain of having mobile MIMO relays, the throughput of systems with minimum overheads is investigated in this section.

A. Individual Mini-slot Throughput

Given a channel matrix at ith BS-relay mini-slot, which is denoted by $\mathbf{H}_{br,i}$, zero-forcing receiver used in the mini-slot gives instantaneous throughput [10]

$$R_{br,i} = L_{br} \sum_{n=1}^{N} \log\Big(1 + \frac{\mathsf{SNR}_{br}/N}{[(\mathbf{H}_{br,i}^{*}\mathbf{H}_{br,i}]_{nn}^{-1}}\Big) \tag{3}$$

bits per symbol, where $\mathbf{H}^{\dagger}$ is the pseudoinverse of $\mathbf{H}$, the superscript $*$ is the complex transpose operation. N is the number of spatial streams, i.e., $\min\{N_{bs}, N_{rn}\}$. $[\mathbf{A}]_{nn}^{-1}$ is a nth diagonal element of the inverse of matrix $\mathbf{A}$. SNR_{br} is the total received SNR at the relay in a linear scale. L_{br} is the time proportion for pure data transmission, which is given by $L_{br} := (T_{br} - V_{su})/T_{br}$ where T_{br} is the mini-slot duration for the BS-relay link in symbols and V_{su} is defined by (1). In addition, denote the expected throughput of ith BS-relay mini-slot by $R_{br} := \mathbb{E}_{\mathbf{H}_{br,i}}[R_{br,i}]$. If $N_{rn} < N_{bs}$, the zero-forcing receiver cannot decode all N_{bs} streams transmitted by the BS. In this case, the BS reduces the number of its transmit streams down to N_{rn} so that it does not overwhelm the relay.

Now consider relay-user mini-slot throughput. Denote a channel vector from the relay to user k in jth mini-slot by $\mathbf{h}_{k,j}$ and its quantized version through feedback by $\hat{\mathbf{h}}_{k,j}$. For the preprocessing, the BS composes channel matrix $\hat{\mathbf{H}}_j = [\hat{\mathbf{h}}_{k_1,j}, \hat{\mathbf{h}}_{k_2,j}, \cdots, \hat{\mathbf{h}}_{k_K,j}]^T$ and computes the preprocessing matrix by $\mathbf{G}_j = \hat{\mathbf{H}}_j^{\dagger} = [\mathbf{g}_{k_1,j}, \mathbf{g}_{k_2,j}, \cdots, \mathbf{g}_{k_K,j}]$. Then, the throughput of the relay-user mini-slot is given by

$$R_{ru,j} = L_{ru}^{(B)} \sum_{k=1}^{K} \log\Big(1 + \frac{\mathsf{SNR}_{ru}|\mathbf{h}_{k,j}^{*}\mathbf{g}_{k,j}|^2}{K + \mathsf{SNR}_{ru}\sum_{p\neq k}|\mathbf{h}_{k,j}^{*}\mathbf{g}_{p,j}|^2}\Big) \tag{4}$$

bits per symbol, where $L_{ru}^{(B)}$ is a time proportion for a relay-user link which is defined by $L_{ru}^{(B)} := (T_{ru} - V_{mu}^{(B)})/T_{ru}$ where T_{ru} is mini-slot duration for a relay-user link. SNR_{ru} is the received SNR at the users, and due to the proximity between relay and users, $\mathsf{SNR}_{ru} \gg \mathsf{SNR}_{br}$.

Let us denote the throughput of the relay-user link with B bits of feedback as $R_{ru,j}^{(B)}$. Also denote the expected throughput of $R_{ru,j}$ and $R_{ru,j}^{(B)}$ as $R_{ru} := \mathbb{E}_{\mathbf{H}_{ru,j}}[R_{ru,j}]$ and $R_{ru}^{(B)} := \mathbb{E}_{\mathbf{H}_{ru,j}}[R_{ru,j}^{(B)}]$, respectively. Then, the following lemma eases the throughput analysis of the relay-user link with limited feedback.

Lemma 1. (Jindal [9]) *Finite feedback with B feedback bits per user incurs a throughput loss relative to perfect CSI at the transmitter zero-forcing upper-bounded by*

$$\frac{1}{L_{ru}^{(B)} \cdot N_{rn}}\big[R_{ru} - R_{ru}^{(B)}\big] < \log\Big(1 + \frac{\mathsf{SNR}_{ru}}{N_{rn}} \cdot 2^{-\frac{B}{N_{rn}-1}}\Big). \tag{5}$$

The throughput for the relay-user link with limited feedback is approximated by using this throughput loss upper-bound as

$$R_{ru}^{(B)} \approx R_{ru} - L_{ru}^{(B)} \cdot N_{ru} \log\big(1 + \frac{\mathsf{SNR}_{ru}}{N_{rn}} \cdot 2^{-\frac{B}{N_{rn}-1}}\big), \tag{6}$$

which is in fact a lower bound for $R_{ru}^{(B)}$.[4] Finally, the throughput for a relay-user link is maximized by $B^* := \arg\max_B R_{ru}^{(B)}$.

B. Frame Throughput

Let us also denote the number of BS-relay mini-slots by n and the number of relay-user mini-slots by m, which altogether form a frame. The scheduler in our system determines these parameters to maximize the expected throughput from the BS to users through relays. Denote the instantaneous frame throughput by R_{prop}. The upper-bound of $\mathbb{E}[R_{prop}]$ is obtained by the following.

Proposition 2. *By scheduling m and n slots for BS-relay and relay-user mini-slots, the expected throughput of the frame from BS to users is upper-bounded by*

$$\mathbb{E}[R_{prop}] \le R_{prop}^{\max} := \frac{R_{br} \cdot R_{ru}^{(B^*)}}{R_{br} + R_{ru}^{(B^*)}}, \tag{7}$$

where the equality holds when $n = R_{ru}$ and $m = R_{br}$.

Proof: Given $\{\mathbf{H}_{br,i}\}_{i=1}^{n}$ and $\{\mathbf{H}_{ru,j}\}_{j=1}^{m}$, the instantaneous throughput of the proposed system is $R_{prop} =$

[4] While the approximation is quite loose for low SNR, thus exaggerating the throughput gains of the proposed system in Section IV, it makes us avoid prohibitive computation for the throughput evaluation with a large number of feedback bits, which is indeed required as will be seen in Fig. 6 in Section IV. Moreover, using this approximation is sufficient to make our point that the overheads are significant in multiuser MIMO downlink system, which is avoidable by MIMO relays that employ single user MIMO.

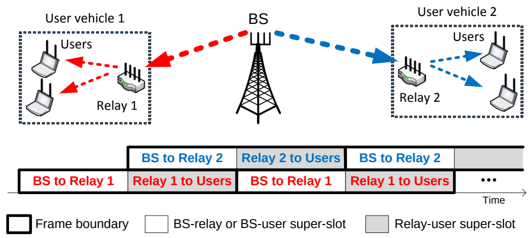

Fig. 3. One example of super-slot schedules for spatial reuse.

$\frac{1}{n+m}\min\{\sum_{i=1}^{n} R_{br,i}, \sum_{j=1}^{m} R_{ru,j}\}$. Since $R_{br,i}$ and $R_{ru,j}$ are iid for i and j, $\mathbb{E}[R_{prop}] \to \frac{1}{n+m}\min\{n \cdot R_{br}, m \cdot R_{ru}\}$ by the law of large number, as n and m grow. Observe that this is maximized to $\frac{R_{br} \cdot R_{ru}^{(B^*)}}{R_{br}+R_{ru}^{(B^*)}}$ by $n = R_{ru}$ and $m = R_{br}$, which completes the proof. ■

By assuming a perfect scheduler for m and n, the upper-bound, $R_{prop}^{\max}$ is studied as the sum throughput of our system.

The sum throughput of the base-line system is computed by (6) with SNR_{br} which is the received SNR from the BS directly to users. While SNR_{ru} is always high due to the proximity between a vehicle and inside users, SNR_{br} may vary as mobile users travel. Interestingly, we can conjecture that (6) is a concave function to B, which means there exists an optimal feedback rate for each SNR that does not necessarily grow when SNR increases. This is against the conclusion made in the previous work [7], [9], which claims that the bits should grow as SNR increases, and will be further investigated in future.

C. Enhancement by Spatial Reuse

Thanks to the asymmetry between a BS-relay and relay-user links, a super-slot for a BS-relay link may be overlapped with that for a relay-user link in time. Observe Fig. 3. Since a relay-user link has far higher channel gain, equivalently higher SNR, the transmission from BS to the relay on the right incurs negligible interference to the relay-user link on the left, and vice versa. This fact is used to schedule super-slots as seen in Fig. 3. We call this scheduling with such consideration as spatial reuse in the proposed system. Moreover, the throughput of a relay-user link is much larger than that of a BS-relay link, which means the BS-relay link is the bottleneck to determine the system throughput. Therefore, with spatial reuse, we have $R_{prop}^{\max} = R_{br}$.

IV. Numerical Results

This section analyzes the numerical results obtained by the throughput analysis in the previous section. Detail channel model and all assumptions are found in Section II-A.

In Figs. 4 and 5, the sum throughput of the proposed relay-assisted system and the baseline system are depicted according to different N_{rn} and to the case whether our system exploits scheduling with spatial reuse (spatial reuse is abbreviated as SR and time sharing is done as TS in the figures). The x-axes of the figures represent the duration of a BS-relay mini-slot, which also represents user mobility: a small mini-slot corresponds to the users with high mobility and vice versa. The duration for a relay-user mini-slot is set to be sufficiently long to have the maximum throughput.

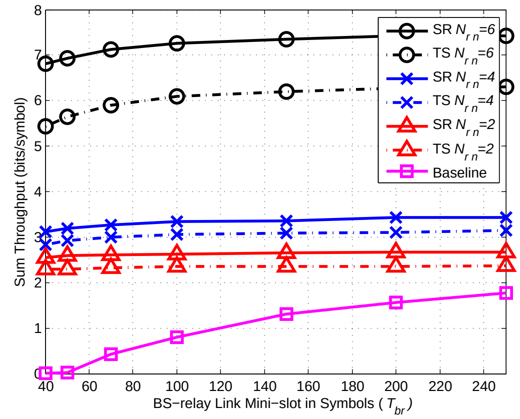

Fig. 4. Sum throughput of the proposed and baseline systems when $\mathsf{SNR}_{bu} = 10$ dB, $N_{bs} = 4$, and $K = 4$. SR is the proposed system with scheduling that considers spatial reuse and TS is the system with scheduling that does not.

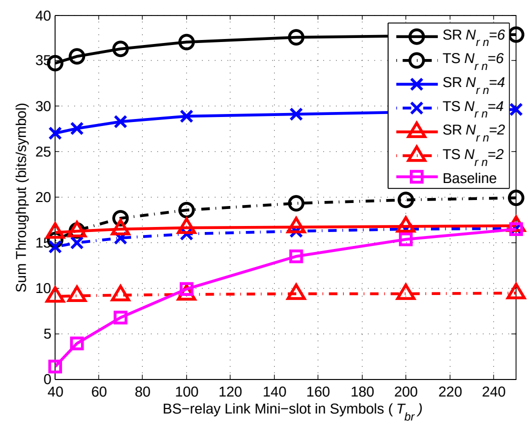

Fig. 5. Sum throughput of the proposed and baseline systems when $\mathsf{SNR}_{bu} = 30$ dB, $N_{bs} = 4$ and $K = 4$.

• *Relay gains:* In Fig. 4, the proposed relay-assisted system achieves higher throughput than the baseline system. The gain over the baseline system is huge especially when a BS-relay mini-slot is small, which indicates the system experiences fast temporal variation of the channel resulted by high mobility. Since the proportion of the overheads to the BS-relay mini-slot duration for our system is less than that of base-line system ($V_{su} < V_{mu}$), our system achieves a large throughput gain with high mobility.

Fig. 5 shows the performance when the distance between BS and users is relatively small, providing high received SNR, 30 dB. It is observed that the baseline system outperforms the proposed one when $N_{bs} > N_{rn}$ and the mini-slot is relatively large. This is because the BS cannot send N_{bs} streams altogether to the relay since it cannot decode the signals due to the lack of antennas. However, with high mobility (equivalently,

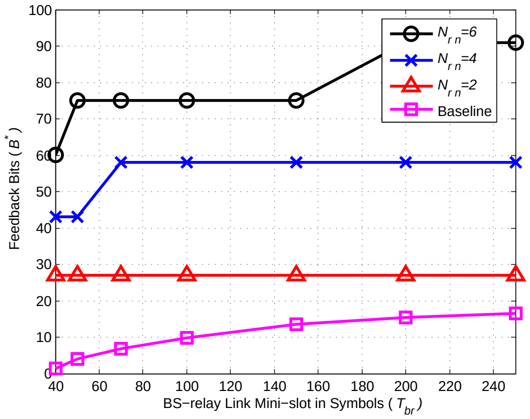

Fig. 6. Bits for CDI feedback when $\mathsf{SNR}_{br} = 30$ dB. There is no difference between the proposed system with spatial reuse or without it.

small BS-relay mini-slot), the proposed system substantially outperforms the baseline system. All these gains come from the reduction of overheads in the proposed system especially for a highly time varying link. Therefore, using MIMO mobile relays makes high throughput communications even with very high mobility.

• *Gains with a relay with more antennas:* Observe dash-lines in both figures, which result by scheduling without spatial reuse. With an aid of additional antennas *at the relay*, the throughput gains are enormous: Fig. 4 shows that having two more antennas at the relay provides up to 3 bits per symbol, which is 100% enhancement. Those antennas add two degrees of freedom in the spatial domain of both BS-relay and relay-user links, substantially improving the quality of MIMO signal processing on both links. This is even with the increase of the feedback overheads in the relay-user mini-slots since the bottleneck link from BS to users is the BS-relay link, not the relay-user link. Note that huge gain is in part because of the sub-optimality of the zero-forcing method [7].

• *Spatial Reuse Gains:* The MIMO relay gains so far come from the additional hardware complexity (at the relay) in the system. The solid lines except for the baseline systems's in Figs. 4 and 5 depict the throughput when spatial reuse in serving relays is perfectly done. Spatial reuse results that the frame is composed of a single BS-relay super-slot even in the proposed system. Through our enhancement by scheduling with spatial reuse, substantial throughput gains are achieved. The gains are magnified by high SNR; the spatial reuse almost doubles the throughput of the system with $N_{rn} = 4$ or $N_{rn} = 6$.

• *Feedback Bits:* Lastly, B^*'s for a relay-user link and for a baseline system are investigated in Fig. 6. For the proposed system, high SNR of a relay-user link requires far more bits for feedback. However, the overhead for such feedback is negligible since the actual data transmission rate is large enough to compensate the loss from it. Therefore, a full channel matrix feedback, which significantly lowers the complexity at the users, may be employed.

V. Discussion

A. Additional Antennas

An alternative solution to enhance the throughput of highly mobile users is to have as many user antennas as those at the BS. Because of form-factor limitation in practice, installation of many antennas at user equipment is not likely to yield a rich scattering that is essential for MIMO operation. Meanwhile, the physical volume of a vehicle can be maximally utilized to install antennas and thus have a desirable MIMO channel. This fact makes the proposed system attractive in practice.

B. Relay Power Control

If transmission from a BS may have a chance to interfere the relay-user link, our system can leverage transmit power control for spatial reuse. Thanks to the high channel gain between a relay and users, the relay may achieve a sufficiently high transmission rate to serve users with low transmit power. Moreover, even when the rate is limited by low transmit power, using multiple mini-slots for a relay-user super-slot can achieve both the completion of its forwarding and the minimization of interference to others.

VI. Conclusion

In this paper, we have proposed a novel transmission strategy for a relay-assisted MIMO system and identified the potential that can enable high throughput mobile wireless access that is multiple times of the system without a relay. By placing a MIMO relay on a vehicle that users are transported by, two asymmetric links are created, and their characteristics are used to reduce the overheads of multiuser MIMO system, which is significant with high mobility. In addition, the asymmetry is further exploited to maximize the spatial reuse of transmissions in our system, doubling the sum throughput when the system is in high SNR regime.

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